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# Re-examination of electroweak symmetry breaking in supersymmetry and implications for light superpartners

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## Abstract

We examine arguments that could avoid light superpartners as an implication of supersymmetric radiative electroweak symmetry breaking. We argue that, from the point of view of string theory and standard approaches to generating the  $\mu$  term, cancellations among parameters are not a generic feature. While the coefficients relating  $M_Z$  to parameters in the soft supersymmetry breaking Lagrangian can be made smaller, these same mechanisms lead to lighter superpartner masses at the electroweak scale. Consequently we strengthen the implication that gluinos, neutralinos, and charginos are light and likely to be produced at the Fermilab Tevatron and a linear collider.

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## 1. Introduction

One of the main successes of supersymmetry is that it can provide an explanation of how the electroweak (EW) symmetry is broken. At a general level three assumptions about the form of the supersymmetric theory are needed for that to work. First, supersymmetry breaking must lead to a soft supersymmetry breaking Lagrangian with mass terms of order a TeV. Second, the  $\mu$  term in the superpotential cannot be a fundamental product of Planck-scale physics but must instead be tied to symmetry breaking at a much lower scale, and third, there must be a quark Yukawa coupling of order unity. The latter two occur naturally in string theories, so when supersymmetry is viewed as

a part of the four-dimensional effective theory following from a more fundamental string theory they are well motivated. The first must remain an assumption until supersymmetry breaking is understood.

For the Higgs potential to actually have a minimum that breaks the EW symmetry two conditions must be satisfied. The one relevant to us here is the only equation that quantitatively relates some soft breaking masses at the electroweak scale to a measured number (at tree level)

$$\frac{M_Z^2}{2} = -\mu^2(\text{EW}) + \frac{m_{H_D}^2(\text{EW}) - m_{H_U}^2(\text{EW}) \tan^2 \beta}{\tan^2 \beta - 1}, \quad (1)$$

where  $m_{H_D}$  and  $m_{H_U}$  are the soft masses for the Higgs doublets coupling to down-type and up-type quarks,

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respectively, and  $\mu$  is the effective  $\mu$  parameter that arises after supersymmetry breaking (we do not give it a separate name). This tree level relation can, in turn, be written in the following way [1]

$$M_Z^2 = \sum_i C_i m_i^2(\text{UV}) + \sum_{ij} C_{ij} m_i(\text{UV}) m_j(\text{UV}). \quad (2)$$

Here  $m_i$  represents a generic parameter of the softly broken supersymmetric Lagrangian at an initial high scale  $\Lambda_{\text{UV}}$  with mass dimension one, such as gaugino masses, scalar masses, trilinear A-terms and the  $\mu$  parameter.

The coefficients  $C_i$  and  $C_{ij}$  depend on the scale  $\Lambda_{\text{UV}}$  and quantities such as the top mass and  $\tan\beta$  in a calculable way through solving the renormalization group equations (RGEs) for the soft supersymmetry breaking terms. For example, taking the running mass for the top quark at the  $Z$ -mass scale to be  $m_{\text{top}}(M_Z) = 170$  GeV, the starting scale to be the grand-unified scale  $\Lambda_{\text{UV}} = \Lambda_{\text{GUT}} = 1.9 \times 10^{16}$  GeV, and  $\tan\beta = 5$  we have for the leading terms in (2)

$$\begin{aligned} M_Z^2 = & -1.8 \mu^2(\text{UV}) + 5.9 M_3^2(\text{UV}) - 0.4 M_2^2(\text{UV}) \\ & - 1.2 m_{H_u}^2(\text{UV}) + 0.9 m_{Q_3}^2(\text{UV}) \\ & + 0.7 m_{U_3}^2(\text{UV}) - 0.6 A_t(\text{UV}) M_3(\text{UV}) \\ & - 0.1 A_t(\text{UV}) M_2(\text{UV}) + 0.2 A_t^2(\text{UV}) \\ & + 0.4 M_2(\text{UV}) M_3(\text{UV}) + \dots, \end{aligned} \quad (3)$$

where the ellipses in (3) indicate terms that are less important quantitatively and for our purposes. In particular  $M_3$  and  $M_2$  are the  $SU(3)$  and  $SU(2)$  soft gaugino masses, respectively, and  $A_t$  is the soft trilinear scalar coupling involving the top squark.  $C_3$  and  $C_\mu$ , being the largest coefficients, are those which we will discuss in some detail below. We think Eq. (2), in a given concrete manifestation such as (3), provides significant insight into high-scale physics whose implications have not yet been fully explored.

Because this equation is the only one connecting supersymmetry breaking to measured data it was long ago realized that it was very important [2–10]. It would be important if (2) could tell us quantitative information about  $M_3$  and  $\mu$ . If  $M_3$  or  $\mu$  is large, and the coefficients  $C_3$  and/or  $C_\mu$  are of order unity or larger, the right-hand side would involve a difference of large numbers, and explanations in physics do not typically involve such fine tuning unless a symmetry

is present. Thus we would tend to expect both  $M_3$  and  $\mu$  to be rather small.

In that case some superpartner masses would be light.  $M_3(\text{UV})$  is closely related to the gluino mass  $M_{\tilde{g}}$ —in the MSSM with  $\Lambda_{\text{UV}} = \Lambda_{\text{GUT}}$  we have  $M_{\tilde{g}} \approx 2.9 M_3(\text{UV})$  at leading order, with an increase from squark and gluino loops of as much as 20%—so naively one would expect the gluino mass to be small enough so that it could be observed at the Tevatron. Similarly,  $\mu$  is in the chargino and neutralino mass matrices, and if it is small some charginos and neutralinos should be light enough to be observed at the Tevatron, even in the limit in which  $M_1$  and  $M_2$  are large. To be somewhat more precise, for this Letter we will define “light superpartner” as one which can be produced at the Tevatron given its energy and expected luminosity. The actual definition would depend on the spectrum, but roughly 600 GeV or less for gluinos and 200 GeV or less for the lighter chargino and neutralinos.

Under what circumstances might such conclusions fail to hold? There are four arguments that have been suggested. First, nature might be unkind and our world may lie at a particular point in the theory where an accidental cancellation occurs. We cannot prove that is not so, of course. But such accidents are rare in physics, and it is appropriate to proceed on the assumption that this does not happen. Second, there could be a relation between  $M_3$  and  $\mu$  in the underlying theory [10], or it could involve some of the other parameters  $m_i$  in the full Eq. (2) with smaller  $C_i$ , such as the Higgs mass  $m_{H_u}^2$  or the other gaugino masses  $M_1$  and  $M_2$ . We will argue below that while not impossible, this outcome is unlikely. Third, the coefficients from the MSSM could change dramatically in extended theories and, in particular, become significantly smaller than unity. Again, we will see below that this is unlikely. What is more, although the coefficients can change, physics that reduces the coefficients (such as beginning the renormalization group evolution of the soft terms from a lower scale) also tends to reduce the physical superpartner masses, so the resulting spectrum is not greatly affected. To our knowledge none of these issues have been systematically addressed previously.

Historically people first examined Eq. (2) with the assumption of gaugino mass degeneracy, in which case the large coefficient of  $M_3$  was taken to also

apply for  $M_1$  and  $M_2$ , leading to predictions of very light charginos and neutralinos that should have been discovered at LEP [3,11,12]. More recently it has been realized that the coefficients of  $M_1$  and  $M_2$ , when treated independently, are quite small and there was no implication that their discovery was expected at LEP [1]. Once  $M_1$  and  $M_2$  are no longer forced to be degenerate with  $M_3$ , electroweak symmetry breaking constrains them very little. Since  $\mu$  is still constrained, however, we will see that light charginos and neutralinos are still expected. Indeed, since the coefficient of  $\mu$  in Eq. (2) is always negative and of order one, and since LEP already constrains  $\mu$  to be larger than about 100 GeV, electroweak symmetry breaking inevitably involves some degree of fortuitous cancellation. Similarly, the large coefficient of  $M_3$  is still of some concern even if light gluinos exist; we will address this issue below.

Once superpartners are discovered one might think that the main implications we are exploring here become less important. Further thought shows that this is not so, since Eq. (2), for a particular choice of  $\tan\beta$  and  $\Lambda_{UV}$  as in (3), then becomes a sum rule giving a constraint on soft breaking parameters which can then help lead us to a deeper understanding of supersymmetry breaking.

## 2. Why $\mu$ and $M_3$ are unlikely to be related in the right way

Naively  $\mu$  and  $M_3$  are unlikely to be related because they seem to arise from different physical mechanisms. Supersymmetry breaking generates  $M_3$ , but additional physics such as the Giudice–Masiero (GM) mechanism [13] or a scalar VEV in the superpotential is needed to generate  $\mu$ . In the former case the  $\mu$  term vanishes in the absence of supersymmetry breaking while in the latter it is often associated with the breakdown of some additional symmetries in the theory unrelated to supersymmetry in a direct way. One could wonder if (say) a stringy approach could produce both a soft supersymmetry breaking parameter  $M_3$  as well as an effective  $\mu$  term in such a way that there is a robust relation between them that could lead to a cancellation in (2) over a range of parameter space.

Since there is no compelling theory of the origin of  $\mu$  it is not clear how to study this issue. Nevertheless,

by examining these well-established approaches to the  $\mu$  problem in the context of string theory we can understand better what physics might affect  $\mu$  and how it might relate to the physics that generates gaugino masses. In fact we learn that rather generally  $\mu$  depends on quite different aspects of the theory from those on which  $M_3$  depends. The remainder of this section is to argue in a number of broad categories that  $\mu$  and  $M_3$  are largely unrelated.

### 2.1. The Giudice–Masiero mechanism in weakly coupled heterotic strings

Let us first examine the possibility of relating  $\mu$  and  $M_3$  in the case of the weakly coupled heterotic string. We follow the approach of Brignole et al. [14] in which one assumes that the communication of supersymmetry breaking from a hidden sector to the observable sector occurs through the agency of one of the moduli fields present in string constructions by the presence of a nonvanishing vacuum expectation value of one or more of their auxiliary fields  $F$ . The nature of the soft supersymmetry breaking terms is then determined by the moduli couplings to observable sector chiral fields. Here we will employ invariance under modular transformation as a guide to constructing these couplings, as in [15].

To be completely general, one can allow for both a superpotential and Kähler potential bilinear in the observable sector fields:

$$\begin{aligned} W(Z^i) &= \frac{1}{2} \sum_{ij} v_{ij}(Z^n) Z^i Z^j + \dots, \\ K(Z^i, \bar{Z}^i) &= \sum_i \kappa_i(Z^n) |Z^i|^2 \\ &\quad + \frac{1}{2} \sum_{ij} [\alpha_{ij}(Z^n, \bar{Z}^{\bar{n}}) Z^i Z^j + \text{h.c.}] + \dots \end{aligned} \quad (4)$$

Here a chiral superfield  $Z$  with a superscript  $m, n$  etc. is supposed to represent a hidden sector field, and we specifically have in mind moduli fields. The chiral fields with superscript  $i, j$  etc. are observable sector fields. The effective  $\mu$  term  $\mu_{ij}$  which arises in the superpotential as a result of (4) for canonically

normalized chiral fields  $\widehat{Z}^i = \kappa_i^{-1/2} Z^i$ , is defined by

$$W(\widehat{Z}^i) \ni \frac{1}{2} \sum_{ij} \mu_{ij} \widehat{Z}^i \widehat{Z}^j + \text{h.c.} \quad (5)$$

and this  $\mu_{ij}$ , which appears in the superpartner mass matrices, is then given by

$$\mu_{ij} = e^{K/2} (\kappa_i \kappa_j)^{-1/2} \tilde{\mu}_{ij}, \quad (6)$$

$$\tilde{\mu}_{ij} = v_{ij} - e^{-K/2} \left( \frac{M}{3} \alpha_{ij} - \bar{F}^{\bar{n}} \partial_{\bar{n}} \alpha_{ij} \right).$$

The terms in (6) involving  $\alpha_{ij}$  are the result of the GM mechanism and depend on the auxiliary fields of the chiral multiplets  $\bar{Z}^{\bar{n}}$  and the auxiliary field of supergravity which is related to the gravitino mass

$$m_{3/2} = -\frac{1}{3} \langle \bar{M} \rangle = \langle e^{K/2} \bar{W} \rangle. \quad (7)$$

We assume throughout, in the manner of [14], that supersymmetry is broken in such a way as to ensure zero vacuum energy at the minimum of the scalar potential.

Modular invariance of the expressions in (4) implies particular functional forms for the  $v_{ij}$  and  $\alpha_{ij}$  which depend on the modular weights of the fields  $Z^i$  involved. Assuming for maximum simplicity that these functions have no dependence on the dilaton  $S$  and that they are both holomorphic in the (overall) modulus field  $T$ , then modular invariance of the Kähler potential and covariance of the superpotential in (4) requires [15]

$$\alpha_{ij}(Z^n) = [\eta(T)]^{-2(n_i+n_j)}, \quad (8)$$

$$v_{ij}(Z^n) = [\eta(T)]^{-2(3+n_i+n_j)},$$

where  $\eta(T)$  is the Dedekind eta function and  $n_i$  is the modular weight of the field  $Z^i$ , defined by  $\kappa_i = (T + \bar{T})^{n_i}$ .

Using the simple tree level Kähler potential  $K = -\ln(S + \bar{S}) - 3\ln(T + \bar{T})$  for the moduli we find in the pure Giudice–Masiero case ( $v_{ij} = 0$ )

$$\mu = m_{3/2} (t + \bar{t})^{-(n_{H_u} + n_{H_d})/2} [\eta(t)]^{-2(n_{H_u} + n_{H_d})} \quad (9)$$

whereas the properly normalized, tree level gaugino mass, under the same assumptions, is given by

$$M_a = \frac{g_a^2(\Lambda_{UV})}{2} F^S + \Delta_a^{\text{loop}} \quad (10)$$

$$= \frac{g_a^2(\Lambda_{UV})}{g_{\text{STR}}^2(\Lambda_{\text{STR}})} \sqrt{3} m_{3/2} \sin \theta + \Delta_a^{\text{loop}}.$$

The second equality holds when the vacuum energy vanishes and we have taken  $\langle s + \bar{s} \rangle = 2/g_{\text{STR}}^2$  where  $g_{\text{STR}}$  is the unified gauge coupling at the string scale  $\Lambda_{\text{STR}}$ . Here  $\theta$  is the familiar Goldstino angle of [14], parameterizing the degree to which the two moduli  $S$  and  $T$  participate in supersymmetry breaking, with  $\theta = 0$  implying moduli domination ( $F^S = 0$ ) and  $\theta = \pi/2$  implying dilaton domination ( $F^T = 0$ ). We have represented the nonuniversal contributions that arise at one loop by  $\Delta_a^{\text{loop}}$ . In string models  $\Delta_a^{\text{loop}}$  will generally be a function of auxiliary fields for the various moduli in the theory as well as the auxiliary field of supergravity [16,17]. These contributions are crucial in creating nonuniversalities in the gaugino sector, but for the sake of our argument in this section we will assume  $\Delta_a^{\text{loop}} = 0$  and merely ask whether  $\mu$  can be related to the *tree* level value of  $M_3$ . Although both  $\mu$  and  $M_3$  vanish when  $M_{3/2}$  does, they depend generically on very different physics and it is not reasonable to assume they are related over a region of the parameters.

The example given above represents the simplest possible outcome from a string perspective as this is sufficient to see the lack of correlation between  $M_3$  and  $\mu$ .<sup>1</sup> Alternatively we can examine the more involved and well-known calculation of  $\mu$  by Antoniadis et al. [20] whose phenomenology was studied in [21, 22]. Bilinear terms in the Kähler potential were identified for (2, 2) compactifications in which a richer moduli spectrum is present. To be as specific as possible, we assume such terms are present for the Higgs fields of the MSSM, which have kinetic terms of the form

$$\kappa_{H_u} = \kappa_{H_d} = \frac{1}{(T + \bar{T})(U + \bar{U})}, \quad (11)$$

where now  $T$  is meant to represent one of the three Kähler moduli and  $U$  is a single complex structure modulus. Under the modified modular symmetries that arise in models with continuous Wilson lines [20,23] the invariant bilinear term is given by

$$\alpha_{H_u H_d} = \frac{1}{(T + \bar{T})(U + \bar{U})}. \quad (12)$$

<sup>1</sup> More complicated cases can be found in the literature, such as allowing the function  $\alpha_{ij}$  to arise through a nonrenormalizable operator that depends on fields charged under an anomalous  $U(1)$  [18,19].

Then the  $\mu$  term is obtained from (6)

$$\mu = \left[ m_{3/2} - \frac{\bar{F}^T}{(T + \bar{T})} - \frac{\bar{F}^U}{(U + \bar{U})} \right] \rightarrow m_{3/2} [1 - \sqrt{3} \cos \theta (\Theta_T + \Theta_U)], \quad (13)$$

where the Goldstino angle  $\theta$  now distinguishes the dilaton from the combined  $T$  and  $U$  sector and  $\Theta_T$  and  $\Theta_U$  represent the individual contributions to SUSY breaking from the particular moduli fields participating in the bilinear term (12) [21,22]. Given that the gluino mass continues to take the form of (10), it is very unlikely that the various moduli VEVs and the pattern of supersymmetry breaking represented by the angles  $\theta$ ,  $\Theta_T$  and  $\Theta_U$  would take precisely the necessary values to maintain a relationship between  $\mu$  and  $M_3$  that could account for the smallness of  $M_Z$ .

## 2.2. Superpotential mechanisms in weakly coupled heterotic strings

The above examples both employed a bilinear in the Kähler potential in the spirit of Giudice and Masiero. One could instead assume that the  $\mu$  term arises from a more complicated effective quadratic term in the superpotential represented by  $v_{ij}$  in (4). For example, consider a higher order nonrenormalizable term in the superpotential where an effective  $\mu$  term is to be generated through the vacuum value of a sequence of fields. Schematically we imagine a superpotential of the form

$$W(Z^i) \sim \frac{Z^{n_1} Z^{n_2} \dots Z^{n_k}}{M_{\text{PL}}^{k-1}} H_u H_d \rightarrow \Lambda_X \left( \frac{\Lambda_X}{M_{\text{PL}}} \right)^{k-1} H_u H_d, \quad (14)$$

where the latter expression is the effective  $\mu$  term when the fields take a VEV  $\langle Z \rangle \sim \Lambda_X$ . If this scale can be engineered to be an intermediate scale such as  $10^{11}$  GeV then a dimension four term would suffice to generate an effective  $\mu$  of about the electroweak scale. This was the basic idea behind the work of [24, 25].<sup>2</sup> Alternatively, if we imagine the fields  $Z^i$  in

(14) to originate from higher order terms in the string compactification then we might imagine the scale  $\Lambda_X$  to be the scale of anomalous  $U(1)$  breaking. Then these VEVs would be much larger since typically  $(\Lambda_X/m_{\text{PL}}) \sim 1/10$  and an electroweak-scale  $\mu$  term would require an operator of rather large dimension.

For anomalous  $U(1)$  breaking we expect the combination of fields cancelling the Fayet–Iliopoulos term to preserve modular invariance [26]. Therefore the canonically normalized effective  $\mu$  term is given by

$$\mu = v \left( \frac{v}{m_{\text{PL}}} \right)^{k-1} \frac{[\eta(t)]^{-2(3+n_{H_u}+n_{H_d})}}{((s + \bar{s})(t + \bar{t}))^{(3+n_{H_u}+n_{H_d})/2}} \quad (15)$$

with  $v$  representing a moduli-dependent generic vacuum value of the scale of the anomalous  $U(1)$  breaking. Again,  $\mu$  depends on physical quantities quite different from those  $M_3$  depends on, since  $M_3$  would continue to be given by (10).

We might alternatively imagine that the  $\mu$  term arises from a renormalizable (dimension three) term in the superpotential such as  $W_N = \lambda_{H_u H_d N}(T) N H_u H_d$ , with  $N$  a singlet under the gauge groups of the Standard Model. Since we are thinking here of possible trilinear terms that are likely to be of fundamental origin from the string theory point of view, we might imagine  $W_N$  coming from an effective  $E_6$  inspired model as might be expected in Calabi–Yau compactifications [2, 27–30], in which case  $W_N$  is accompanied by additional trilinear terms coupling the Standard Model singlet to exotic states such as vector like triplets of  $SU(3)$   $D$  and  $\bar{D}$ . The details of how a vacuum value  $\langle \lambda_{H_u H_d N}(T) N \rangle \simeq 1$  TeV is arranged is immaterial for our purposes, however. It is sufficient to note that if the Yukawa coupling  $\lambda_{H_u H_d N}$  is allowed by the string selection rules (which will often be the case since such a coupling is allowed by  $E_6$  gauge invariance where  $N$  is interpreted as a singlet under the  $SO(10)$  subgroup of  $E_6$ ), then it is presumably a fundamental (tree level) string term and modular invariance of the superpotential term  $W_N$  is to be expected. We are thus led to an

<sup>2</sup> Though in that case the bilinear quantity receiving a VEV was assumed to be a quark–antiquark bilinear  $Q\bar{Q}$  from the hidden sector, so it represents a nonrenormalizable operator stemming from

the low-energy field theory as opposed to one originating from the underlying string theory, where we would expect a holomorphic term such as the one in (14).

effective  $\mu$  term

$$\mu = \lambda_{H_u H_d N} \langle N \rangle \times \frac{[\eta(T)]^{-2(3+n_{H_u}+n_{H_d}+n_N)}}{((s+\bar{s})(t+\bar{t}))^{(3+n_N+n_{H_u}+n_{H_d})/2}}, \quad (16)$$

where the constant  $\lambda_{H_u H_d N}$  is presumably  $\mathcal{O}(1)$ . Again, the gluino mass is still given by (10). In both of these superpotential examples the magnitude of the effective  $\mu$  is related to the breaking of some symmetry (an anomalous  $U(1)$  in the first example, a  $U(1)'$  arising from the breaking of  $E_6$  to the Standard Model in the second) not directly related to supersymmetry. Even if the initial challenge of arranging these symmetry-breaking scales properly to ensure  $\mu \sim \mathcal{O}(M_{3/2})$  can be overcome, we still are no better off than in the Kähler potential cases studied previously.

### 2.3. Beyond the weakly coupled heterotic string

So far the discussion has been couched in the context of weakly coupled heterotic string theory compactified at a high scale  $\Lambda_{\text{STR}} \simeq M_{\text{PL}}$ , with supersymmetry breaking transmitted via moduli to the observable sector at a slightly lower scale  $\Lambda_{\text{UV}} \simeq \mathcal{O}(10^{16} - 10^{17})$  GeV. Nevertheless our conclusions continue to hold in the strongly-coupled heterotic limit of M-theory in the absence of nonperturbative objects such as five-branes. In such cases the tree level gaugino masses are modified from their relation in (10) to the form [31–34]

$$M_a = \frac{g_a^2(\Lambda_{\text{UV}})}{g_{\text{STR}}^2(\Lambda_{\text{STR}})} \frac{\sqrt{3} m_{3/2}}{1 + \epsilon} \left[ \sin \theta + \frac{1}{\sqrt{3}} \epsilon \cos \theta \right], \quad (17)$$

where  $\epsilon$  is a parameter which is zero in the weak-coupling limit and we have suppressed the possible relative phase between the two terms. We continue to expect the  $\mu$  term to be generated by one of the mechanisms stated above, as was considered in [34].

Meanwhile recent developments in string theory have led to a great deal of research on Type I/Type IIB string theories, with fundamental string scales that can be significantly lower than the Planck scale. This activity has thus far provided few new insights, however, into the origin of the  $\mu$  term and its relation to soft supersymmetry breaking parameters such as  $M_3$ . For

example, phenomenological studies of D-brane models constructed directly from string theory, with an intermediate string scale  $\Lambda_{\text{STR}} \sim 10^{11}$  GeV, have generally left  $\mu$  unspecified [35–38], allowing us to pick the mechanism above that suits our fancy. As these examples continue to imagine SUSY breaking communicated from a hidden brane to our observable brane via moduli, we can continue to use the framework of [14] to explore the soft terms in such theories.

Thus we see very generally that a relation between  $\mu$  and  $M_3$  that gives a nonaccidental cancellation seems very unlikely in string-derived models with intermediate to high string scales. Furthermore, in this section we have restricted ourselves to the simplest and most conservative formulae, ignoring effects such as expected gaugino mass nondegeneracies at the loop level or the presence of additional moduli which participate in supersymmetry breaking, which would only strengthen our conclusions.

Ideally we would survey all models in the literature in an attempt to confirm that  $M_3$  and  $\mu$  are never related in just such a way as to have small fine tuning in the electroweak sector. Even apart from the difficulty of such a task, many models of very low-energy strings or brane-world constructions are not formulated with sufficient precision to judge their fine-tuning implications. Often it is considered sufficient to merely obtain  $\mu \sim m_{3/2} \sim m_{1/2}$  to avoid fine-tuning and hence be considered “natural”. However it is difficult to imagine such an imprecise relation guaranteeing small cancellations in (2) and (3) over a range of the free parameters in such models, as we have explained in the context of more precise cases examined above. In the context of our current understanding of the  $\mu$ -problem in heterotic string-based models a robust relation between  $\mu$  and  $M_3$  is hard to fathom, though this may reflect a lack of understanding of the fundamental physics behind supersymmetry breaking.<sup>3</sup> To put it differently, if there exists a model in which certain soft parameters and  $\mu$  are related *of necessity* in just the needed manner as to have negligible amounts of fine-tuning in the

<sup>3</sup> See, however, the results of [39,40] for interesting ways in which this problem can be addressed outside of the effective supergravity theories based on the heterotic string.

electroweak sector, then this model should be taken very seriously.

### 3. Why $M_3$ is unlikely to be related to other soft terms in the right way

The main conclusion to be drawn from the previous section is that, at a minimum, the value of  $M_Z$  depends on two unrelated scales: the gross scale of the soft supersymmetry breaking terms, characterized by  $m_{3/2}$ , and the scale of symmetry breaking that determines  $\mu$ . Even when these two gross scales are related, as in the mechanism of Giudice and Masiero, the details of the physics determining  $M_3$  and  $\mu$  suggest that obtaining relative magnitudes for these parameters that are related in such a way as to allow for their absolute magnitudes to be large without concomitant large cancellations would be an accidental outcome of an underlying theory.

But assuming, for the moment, that such a theory is found—can we still find relations *among* the soft terms in the Lagrangian themselves such that a heavy gaugino sector need not imply large cancellations in (2)? After all, the soft Lagrangian is grossly defined by only one parameter,  $m_{3/2}$ . For example, the simplest and most constrained limit of the weakly coupled heterotic string is the so-called “dilaton dominated” limit in which only the dilaton plays a role in transmitting supersymmetry breaking from the hidden sector to the observable sector. In that case we most certainly do have a relation between gaugino and scalar masses, namely  $M_a = \sqrt{3}m_0$ , where both the scalar masses and gaugino masses are unified at the GUT scale. And this is a “robust” relation in the sense that it does not depend on certain other parameters in the theory, such as the vacuum values of string moduli like  $S$  and  $T$ .

So relations among soft terms are more likely to occur than between  $\mu$  and  $M_3$ , but are they likely to be both robust and also of the sort to allow large values for these soft terms, relative to  $M_Z$ , without large cancellations? We first consider the special role in Eq. (2) played by  $m_{H_U}^2$  in determining the  $Z$  mass. The coefficient  $C_{H_U}$  is both of the right sign and general magnitude to provide cancellation against  $C_3$ . But note that in the case of universal scalar masses this feature disappears. Taking all scalar masses in (3) to be

given by  $m_0$  we would have

$$M_Z^2 = -1.8\mu^2(\text{UV}) + 0.4m_0^2 + 5.9M_3^2 - 0.4M_2^2 + \dots \quad (18)$$

so this mechanism of making large values of  $M_3$  natural requires nonuniversal scalar masses. In particular the Higgs masses must be divorced from the scalar masses of the matter fields of the MSSM.<sup>4</sup>

The tree level gaugino masses and scalar masses in a general supergravity theory are given by

$$M_a^0 = \frac{g_a^2}{2} F^n \partial_n f_a^0, \quad (19)$$

$$(m_i^0)^2 = \left\langle \frac{M\bar{M}}{9} - F^n \bar{F}^{\bar{m}} \partial_n \partial_{\bar{m}} \ln \kappa_i \right\rangle, \quad (20)$$

where  $f_a^0 = S$  in the weakly-coupled limit,  $M$  is the supergravity auxiliary field from (7) and  $\kappa_i$  was defined in (4). Tree level nonuniversality could, in principle, arise from differing moduli-dependence of the Kähler metrics  $\kappa_i$  of the Higgs fields from the rest of the observable sector, say via differing modular weights  $n_i$ . Alternatively, we could imagine cases in which the tree level scalar mass in (20) is precisely zero, as in models with a no-scale structure. Then the leading scalar masses arise through loop effects and will necessarily be nonuniversal. Each of these outcomes is possible, but the theory must now arrange for the contributions to each of the individual scalar masses, as well as the gluino mass, to be of such a magnitude as to allow *automatic* cancellations among these soft terms—and thereby allow these parameters to be much larger than  $M_Z$  without fine-tuning through large cancellations. The question again becomes model-dependent.<sup>5</sup>

From the string theory perspective, even in the simplest case of the weakly-coupled heterotic string, the soft Lagrangian is in general determined by *three* independent scales as opposed to simply  $m_{3/2}$ . The first of these is the overall scale of supersymmetry break-

<sup>4</sup> In our analysis of Section 4 below we will also see that the magnitude of  $C_{H_U}$  is also unique in that its coefficient remains quite constant, even in extended theories. This is a manifestation of the same physics behind the “focus point” effect in its running, as noted in [41].

<sup>5</sup> For an analysis of fine-tuning in the case of nonuniversal Higgs soft masses, see [42].

ing given by  $M \sim m_{3/2}$  and appearing as the leading term in the soft scalar masses (20). The other two scales depend on the degree to which the two broad classes of string moduli (the dilaton and the compactification moduli such as the  $T$  and  $U$ ) participate in the transmission of this supersymmetry breaking to the observable sector, via nonzero auxiliary fields  $F$ . The gaugino masses will be determined, at leading order in weak coupling, by the dilaton contribution. By contrast, scalar masses feel the effects of moduli auxiliary fields through the moduli dependence of the  $\kappa_i$ , which tend to include the compactification moduli. This suggests that each of these three scales must be engineered to be of the right proportions to one another. Then we return to the case of  $\mu$  and  $M_3$  where the smallness of  $M_Z$  can be maintained in the face of large values for the soft supersymmetry breaking gaugino masses, without the implication of large cancellations in (2), only provided certain precise model-dependent relations hold.

To summarize, cancellations of some sort must clearly occur in (2) in order to obtain  $M_Z = 91$  GeV. This is not in dispute. Furthermore, every reasonable model of physics at the supersymmetry breaking scale must make predictions for the soft terms and  $\mu$  which will imply certain relations among them that depend on internal model dynamics. But the larger each individual term becomes in a particular expression such as (3) or (18), the more precisely those internal parameters must be specified to avoid an unreasonable cancellation in the determination of the  $Z$  mass. Alternatively, the smaller the individual soft terms are, in particular the value of  $\mu$  and  $M_3$ , the less we must rely on such precise relations and the less “accidental”  $M_Z = 91$  GeV becomes.

#### 4. The coefficients of $\mu$ and $M_3$

Accepting the premise, then, that the observed magnitude of the  $Z$  mass is not an accidental outcome of an underlying theory, we now seek to investigate the strength of this argument to changes in the framework used to obtain expressions such as (3). Since the coefficients (fine-tuning parameters) in (2) are obtained from the input parameters by solving the renormalization group equations (RGEs) and running from the input scale down to the electroweak scale, their val-

ues could potentially depend on the assumptions we make in solving the RGEs. We now study the effects of changing some of those assumptions. In what follows we will work with one loop RGEs and solve the tree level electroweak symmetry breaking conditions. Including the full one loop radiative corrections to the Higgs potential tends to reduce the fine-tuning, particularly for small  $\tan \beta$ , but will not change the conclusions we will draw in this section.

##### 4.1. The dependence on the choice of $\tan \beta$ , $\lambda_{\text{top}}$ and input scale

One of the key ingredients of radiative electroweak symmetry breaking is the existence of a Yukawa coupling of  $\mathcal{O}(1)$ . For a large region of the parameter space, the large Yukawa coupling is provided by the top quark Yukawa  $\lambda_t$ . Indeed, the nature of the electroweak symmetry breaking (i.e., the values of the fine-tuning coefficients  $C_i$ ) is quite sensitive to the choice of  $\lambda_t$ , as is well known [3–6]. Of course this choice depends on the value of  $\tan \beta$  and  $\lambda_t$ . We studied the effect of  $\lambda_t$  on the fine-tuning coefficients  $C_i$  for the leading terms  $C_3$  and  $C_\mu$ . Fig. 1 plots the values of these two coefficients as a function of  $\tan \beta$  for a running top mass at the  $Z$ -mass of  $m_{\text{top}}(M_Z)$  of 164 GeV and  $\Lambda_{\text{UV}} = \Lambda_{\text{GUT}}$ . In all of the analysis that follows we work with the value  $\alpha_3(M_Z) = 0.119$  for the strong coupling constant. The values of  $C_3$  and

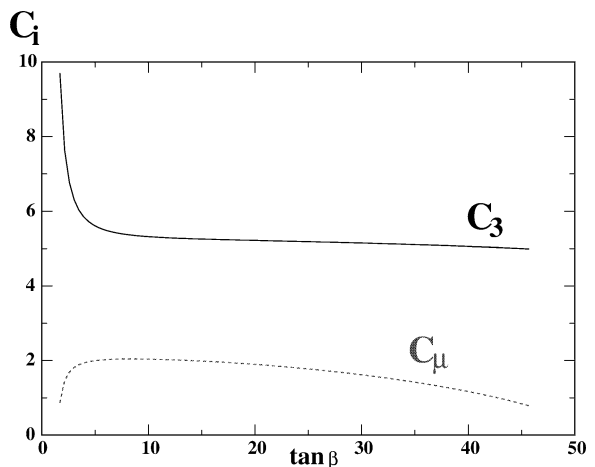


Fig. 1. The dependence of the fine-tuning coefficients  $C_3$  and  $C_\mu$  on the value of  $\tan \beta$  for a running top quark mass at  $M_Z$  of  $m_{\text{top}}(M_Z) = 164$  GeV.



$C_\mu$  are most sensitive at low values of  $\tan\beta$  and are largely independent of  $\tan\beta$  for  $\tan\beta \gtrsim 4$  and below its perturbative limit.

In Fig. 2 we choose two values of  $\tan\beta$  and investigate the importance of  $m_{\text{top}}$  on the same coefficients for an initial scale  $\Lambda_{\text{UV}} = \Lambda_{\text{GUT}}$ . It has long been appreciated that fine-tuning related to the  $\mu$  parameter was relaxed for larger values of  $m_{\text{top}}$  and larger values of  $\tan\beta$ , as is borne out by Figs. 1 and 2. What is not

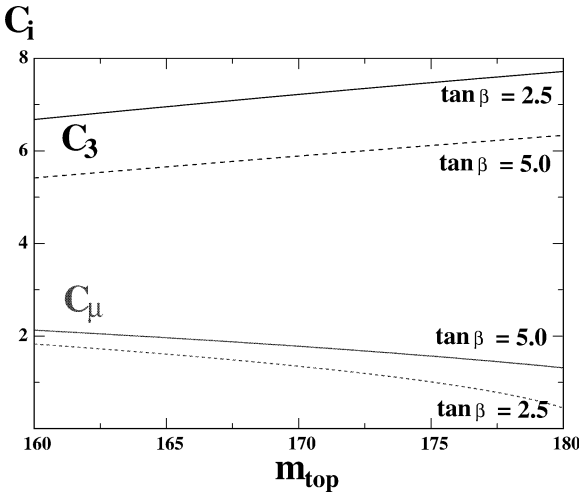


Fig. 2. The dependence of the fine-tuning coefficients  $C_3$  and  $C_\mu$  on the value of the running top quark mass  $m_{\text{top}}(M_Z)$ , for  $\Lambda_{\text{UV}} = 3 \times 10^{16}$  GeV.

often appreciated is that these same directions tend to *increase* the fine-tuning related to the gluino mass  $M_3$ .

The usual assumption is to run the RGEs from the scale where the Standard Model gauge couplings unify,  $\Lambda_{\text{GUT}}$ . However, it is typical that supersymmetry breaking actually appears in the observable sector at a lower scale, such as  $\Lambda_{\text{INT}} \sim 10^{11} - 10^{14}$  GeV in models with intermediate string scales, or  $\Lambda_{\text{GMSB}} \sim 10^5 - 10^8$  GeV in gauge-mediated models. In these cases the input scale for the soft parameters  $\Lambda_{\text{UV}}$  should be identified as the lower supersymmetry breaking scale. Generally speaking, lowering the input scale can have sizable effects on the values of some of the fine-tuning coefficients.

In Table 1 we display the leading coefficients among the  $C_i$  and  $C_{ij}$  of (2) as a function of the input scale  $\Lambda_{\text{UV}}$  for five different choices of high energy scale, where we have set  $m_{\text{top}}(M_Z) = 170$  GeV and  $\tan\beta = 5$  throughout. Note that the coefficient  $C_\mu$  in front of  $\mu^2(\text{UV})$  is essentially independent of the input scale while the coefficient  $C_{H_u}$  grows with lower input scale—eventually supplanting  $C_3$  as the most important soft mass in determining  $M_Z$ . Given that the sign of  $C_{H_u}$  and  $C_\mu$  are the same, Table 1 suggests that provided  $m_{H_u}^2(\text{UV})$  is positive at the initial SUSY breaking scale the overall problem of unnatural cancellations in (2) is worse in lower-scale SUSY breaking mechanisms.

Table 1

Leading coefficients  $C_i$  and  $C_{ij}$  for  $M_Z^2$  in (2) as a function of  $\Lambda_{\text{UV}}$  for  $M_{\text{top}} = 170$  GeV and  $\tan\beta = 5$

	Case 1.A	Case 1.B	Case 1.C	Case 1.D	Case 1.E
$\Lambda_{\text{UV}}$ (GeV)	$1 \times 10^{16}$	$1 \times 10^{14}$	$1 \times 10^{11}$	$1 \times 10^8$	$1 \times 10^5$
$M_1^2(\text{UV})$	0.001	-0.006	-0.01	-0.02	-0.01
$M_2^2(\text{UV})$	-0.4	-0.3	-0.2	-0.3	-0.2
$M_3^2(\text{UV})$	5.9	4.4	2.6	1.3	0.4
$A_{\text{top}}^2(\text{UV})$	0.2	0.2	0.2	0.3	0.2
$\mu^2(\text{UV})$	-1.8	-1.8	-1.7	-1.7	-1.8
$m_{Q_3}^2(\text{UV})$	0.9	0.8	0.7	0.6	0.4
$m_{U_3}^2(\text{UV})$	0.7	0.7	0.6	0.5	0.4
$M_2(\text{UV})M_3(\text{UV})$	0.4	0.3	0.2	0.08	0.02
$A_t(\text{UV})M_3(\text{UV})$	-0.6	-0.6	-0.5	-0.4	-0.2
$A_t(\text{UV})M_2(\text{UV})$	-0.1	-0.1	-0.1	-0.07	-0.03

#### 4.2. The dependence on possible matter at intermediate scales

Usually, studies of radiative electroweak symmetry breaking are based on the assumption that the MSSM is valid from the electroweak scale  $\Lambda_{EW}$  all the way up to  $\Lambda_{UV} \simeq \Lambda_{GUT}$ . On the other hand, many models constructed from some fundamental theory contain a number of fields in addition to the MSSM matter content. While it is quite possible that all of these fields decouple at  $\Lambda_{UV}$ , in which case they would have no effect on the discussion of fine-tuning described above, it is also plausible that some of them may acquire masses at some intermediate scale below the SUSY breaking scale  $\Lambda_{EW} < \Lambda_I < \Lambda_{UV}$ . The intermediate scale matter will then give threshold corrections which alter the running of the soft parameters and can affect the values of the corresponding fine-tuning coefficients.

Let us consider this question in a general context before investigating specific cases. Clearly the importance of the gluino mass  $M_3$  in (2) is a manifestation of its importance in the RG evolution of the soft parameters entering into (1). Consider for a moment the system of equations given in schematic form by

$$\frac{dm_{H_u}^2}{dt} \sim \frac{1}{16\pi^2} [|\lambda_t|^2 (m_{H_u}^2 + m_Q^2 + m_U^2) + |A_t^2|^2] \dots, \quad (21)$$

$$\frac{d\lambda_t}{dt} \sim \frac{\lambda_t}{16\pi^2} \left[ |\lambda_t|^2 - \frac{16}{3} g_3^2 \right] + \dots, \quad (22)$$

$$\frac{dA_t}{dt} \sim \frac{1}{16\pi^2} \left[ A_t \left( |\lambda_t|^2 - \frac{16}{3} g_3^2 \right) + \lambda_t \left( \frac{32}{3} g_3^2 M_3 \right) \right] + \dots, \quad (23)$$

$$\frac{dm_{\tilde{q}}^2}{dt} \sim \frac{1}{16\pi^2} \left[ m_{\tilde{q}}^2 |\lambda_t|^2 - \frac{32}{3} g_3^2 |M_3|^2 \right] + \dots, \quad (24)$$

where we have kept only the leading terms proportional to top-quark Yukawas or  $g_3$ . We have also introduced the running scale  $t = \ln(\Lambda/M_Z)$ . In Eq. (24)  $\tilde{q}$  could be any scalar quark soft mass.

A few comments are in order. The dependence of  $m_{H_u}^2$  (EW) on  $M_3^2$  (UV) comes from the strong  $M_3^2$  dependence of the running of soft parameters such as  $m_{\tilde{q}}^2$  and  $A_t$ . Adding intermediate scale matter charged under color will affect the RGE evolution of the above

quantities, and hence the fine-tuning coefficient of  $M_3$ . The running of the Yukawa coupling  $\lambda_t$  is also largely controlled by the  $g_3^2$  term in its RGE. Therefore, threshold corrections to the running of  $g_3^2$  will have a large effect on the value of  $\lambda_t^2$  (UV) which in turn has a significant impact on the fine-tuning coefficients.

The intermediate scale matter can also be charged under  $SU(2)$  and couple to MSSM up-type Higgs with a new Yukawa coupling. If such a coupling exists it will lead to extra terms in the RGE of  $m_{H_u}^2$ . Let  $X_q$  represent new chiral fields which are doublets under  $SU(2)$  and triplets under  $SU(3)$  and let  $X_l$  represent new chiral superfields which are doublets under  $SU(2)$  alone. Then if couplings with  $H_u$  exist we expect new terms in the RGE for  $m_{H_u}^2$  of the heuristic form

$$\delta \left( \frac{dm_{H_u}^2}{dt} \right) = \theta_{X_q}(t) |\lambda_{X_q}|^2 m_{X_q}^2 + \theta_{X_l}(t) |\lambda_{X_l}|^2 m_{X_l}^2. \quad (25)$$

The  $\theta$ 's in (25) are properly defined step functions turning on the couplings above the energy scale  $\Lambda_I$ . We must distinguish, then, the intermediate matter based on their charge under  $SU(3)$  versus  $SU(2)$ . The running of any new “squarks”  $m_{X_q}^2$  will necessarily introduce new positive contributions into the running of  $m_{H_u}^2$  due to the  $M_3^2$  term in its RGE. This will in turn enhance the fine-tuning parameter  $C_3$ .<sup>6</sup> On the other hand, adding new “leptons”  $m_{X_l}^2$  as in the second term in (25) will not bring in large new contributions to the running of  $m_{H_u}^2$  because their RGEs are independent of the gluino at leading order. An immediate conclusion one can draw from the discussion here is that if there is additional matter at some intermediate scale charged under color with a Yukawa coupling to the MSSM Higgs, those Yukawa couplings will be strongly constrained by the naturalness requirement of the electroweak symmetry breaking.

Next, we turn to study the effects of intermediate scale matter on the value of  $C_\mu$ . At one loop, the RGE

<sup>6</sup> Notice that a sizable Yukawa coupling between some heavy fields charged under color to the ordinary Higgs is the necessary and sufficient condition for an  $\mathcal{O}(1)$  enhancement of the fine-tuning. It does not depend on the initial value of  $m_{X_q}^2$  because the running of  $m_{X_q}^2$  will contain a large contribution of  $M_3$  (UV) regardless of its initial value.

of  $\mu$  in the MSSM is

$$\frac{d\mu}{dt} = \frac{\mu}{16\pi^2} \left[ 3|\lambda_t|^2 + 3|\lambda_b|^2 + |\lambda_\tau|^2 - 3g_2^2 - \frac{3}{5}g_1^2 \right]. \quad (26)$$

First, we notice that since  $\mu$  only receives wavefunction renormalizations, the solution of its RGE will always have the form  $\mu(t) = \mu(\text{UV})f(t, \dots)$ , where  $f$  is a function of scale and other parameters, and therefore  $C_\mu \sim |f|^2$ . Positive terms on the right-hand side of (26) proportional to the square of the magnitudes of Yukawa couplings will therefore cause  $C_\mu$  to grow and negative ones proportional to the squares of the gauge couplings will cause it to diminish. This is the origin of the observed behavior of the coefficients  $C_3$  and  $C_\mu$  in the context of the MSSM: stronger running of  $g_3$  is correlated with lower  $C_3$  but larger  $C_\mu$ .

The existence of potential intermediate scale matter can have several types of effects on the running of  $\mu$ , beyond the threshold corrections to the beta functions of the gauge couplings. If the intermediate scale matter couples to the MSSM Higgs through some Yukawa type couplings  $\lambda_X$  then there will be an additional effect on the running of  $\mu$ . Those corrections always have the form of

$$\delta\left(\frac{d\mu}{dt}\right) = \theta(t) \sum_X |\lambda_X|^2, \quad (27)$$

where  $\theta(t)$  is zero below the intermediate scale  $\Lambda_I$  and  $\sum_X \lambda_X$  represents the sum of all such new Yukawa couplings.<sup>7</sup> Generically, we see that the inclusion of extra states with nonvanishing Yukawa couplings to the Higgs sector can make  $C_\mu$  smaller.

The study of the solutions of the RGEs above helps to illuminate the origin of the fine-tuning coefficients. Next we will use this insight to study the possibility of suppressing the fine-tuning parameters  $C_3$  and  $C_\mu$  through judicious choices of intermediate scale matter combinations and couplings. We begin with the change of the beta functions of the gauge couplings from the introduction of intermediate scale matter. The

beta function coefficients of the MSSM, in their GUT normalization, are given by

$$\begin{aligned} \frac{dg_a}{dt} &= -\frac{b_a g_a^3}{16\pi^2}, \\ b_1 &= -\frac{33}{5}, \quad b_2 = -1, \quad b_3 = 3. \end{aligned} \quad (28)$$

Above the intermediate scale  $\Lambda_I$  they will be modified to  $b'_i = b_i + \delta_i$ . For simplicity let us initially choose  $\delta_1 = \delta_2 = \delta_3 = \delta$ , which would represent new matter in complete multiplets of  $SU(5)$  which preserves gauge coupling unification at the usual GUT scale  $\Lambda_{\text{gut}} \simeq 2 \times 10^{16}$  GeV.

We can achieve a reduction in  $C_3$  (at the expense of a higher value of  $C_\mu$  as mentioned above) by putting in a negative  $\delta$  through the addition of new fundamentals charged under the Standard Model. To be concrete we can take as an example the states coming from a chiral supermultiplet of representations **27** and  $\overline{\mathbf{27}}$  of  $E_6$ . Under the decomposition under the Standard Model gauge group those states are:  $(\mathbf{2}, \mathbf{1}^c)_{-1}$ ,  $(\mathbf{1}, \mathbf{1}^c)_2$ ,  $(\mathbf{1}, \mathbf{3}^c)_{-\frac{4}{3}}$ ,  $(\mathbf{1}, \overline{\mathbf{3}}^c)_{\frac{2}{3}}$  and  $(\mathbf{2}, \mathbf{3}^c)_{\frac{1}{3}}$ , plus their complex conjugate representations [43]. Here the numbers in the parenthesis label the representations under  $SU(2)$  and  $SU(3)$  while the lower indices outside are the hypercharges. One copy of them at some intermediate scale will give rise to a total threshold correction  $\delta_1 = \delta_2 = \delta_3 = \delta = -4$ . We can also have states coming from vector supermultiplets of the adjoint representation of  $E_6$ . They decompose similarly under the Standard Model gauge group as the first five of the states coming from the chiral supermultiplet. The threshold correction of one copy of those states will be  $\delta = 12$ .

As we mentioned above, the inclusion of this type of threshold correction will induce some enhancement in the fine-tuning coefficient  $C_\mu$  associated with  $\mu$ . In Table 2 we demonstrate this fact by presenting the leading coefficients  $C_i$  and  $C_{ij}$  for different sets of intermediate scale matter, including a case where the additional fields do not come in complete multiplets of  $SU(5)$  but are instead designed to yield gauge coupling unification at the string scale [43,44]. We have begun the running of all soft terms, as well as the  $\mu$  parameter, from the scale  $\Lambda_{\text{UV}}$  which we have taken to coincide with the scale of gauge coupling unification in each case. The values in Table 2 do not

<sup>7</sup> We will later be interested in particular Yukawa couplings which do not couple to fields charged under color since otherwise they will enhance the fine-tuning significantly. An example might include a Yukawa coupling between a heavy right-handed neutrino to a left-handed neutrino and Higgs providing the off-diagonal elements in the see-saw mass matrix.

Table 2

Leading coefficients  $C_i$  and  $C_{ij}$  for  $M_Z^2$  as in (2) for various intermediate matter scenarios. Case (A) has a common delta of  $-12$ . Case (B) has a common delta of  $-4$ . Case (C) has  $\delta_1 = -1$ ,  $\delta_2 = -3$  and  $\delta_3 = -4$ , which represents the case where we have added two pairs of  $(D, \bar{D})$  and one pair of  $(Q, \bar{Q})$ . This table assumes a top mass  $m_{\text{top}}(M_Z) = 164$  GeV and  $\tan \beta = 5$

	Case 2.A	Case 2.B	Case 2.C
$\Lambda_{\text{UV}}$ (GeV)	$3 \times 10^{16}$	$3 \times 10^{16}$	$4 \times 10^{17}$
$\Lambda_{\text{int}}$ (GeV)	$5 \times 10^{11}$	$5 \times 10^{11}$	$1 \times 10^{14}$
$\alpha_{\text{GUT}}(\Lambda_{\text{UV}})$	0.29	0.06	0.05
$M_2^2(0)$	$-0.05$	$-0.2$	$-0.3$
$M_3^2(0)$	0.6	3.3	4.3
$A_{\text{top}}^2(0)$	0.2	0.2	0.2
$\mu^2(0)$	$-2.9$	$-2.2$	$-2.2$
$m_{Q_3}^2(0)$	0.8	0.8	0.8
$m_{U_3}^2(0)$	0.6	0.6	0.6
$m_{H_u}^2(0)$	$-1.3$	$-1.3$	$-1.4$
$M_2(\text{UV})M_3(\text{UV})$	0.07	0.3	0.4
$A_t(\text{UV})M_3(\text{UV})$	$-0.3$	$-0.6$	$-0.6$
$A_t(\text{UV})M_2(\text{UV})$	$-0.1$	$-0.1$	$-0.2$

assume any additional Yukawa couplings between the new matter multiplets and the fields of the Standard Model. As a consequence the reduction in  $C_3$  is always related to an increase in  $C_\mu$ .

However, this increase in  $C_\mu$  can be mitigated by introducing some extra Yukawa couplings associated with some heavy field, as in (26), while being sure not to allow sizable Yukawa interactions between the new heavy fields which are charged under color as this will induce new contributions to the fine-tuning of  $M_3$  through the RGE of  $m_{H_u}^2$ . In Fig. 3 we show the resulting reduction in both  $C_3$  and  $C_\mu$  with two different values of  $\delta$ . For both cases we assume a set of additional Yukawa couplings  $\lambda_X$  between the new heavy color-singlet states that appear at the scale  $\Lambda_I$  and the Higgs fields of the Standard Model. For concreteness we have chosen  $\sum_X |\lambda_X|^2 = 5$  so as to maximize the effect.

A simultaneous reduction in both  $C_3$  and  $C_\mu$  which preserves gauge coupling unification would require a special combination of exotic states with (a) some being charged under  $SU(3)$  of the Standard Model to reduce  $C_3$  but no Yukawa couplings to the MSSM Higgs sector and (b) additional states which are sin-

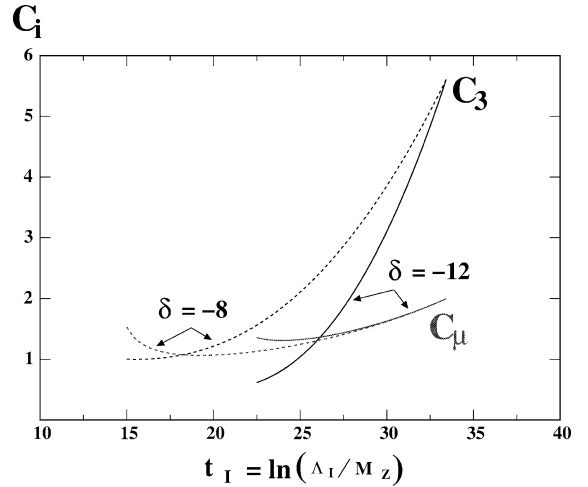


Fig. 3. The dependence of the fine-tuning coefficients  $C_3$  and  $C_\mu$  on threshold corrections and the location of the intermediate scale  $\Lambda_I$ . The solid curves are for a common  $\delta = -12$  and corresponding  $\alpha_{\text{GUT}}(\text{UV}) = 0.29$ , while the dashed curves are for a common  $\delta = -8$  and corresponding  $\alpha_{\text{GUT}}(\text{UV}) = 0.06$ . This plot is for  $\tan \beta = 5$ ,  $m_{\text{top}}(M_Z) = 164$  GeV and assumes a new set of Yukawas such that  $\sum_X |\lambda_X|^2 = 5$ .

glets under  $SU(3)$  with generally large Yukawa couplings to the Higgs sector to reduce  $C_\mu$ .

Table 3

Electroweak scale running gluino mass for equivalent cancellations in (2). The last column shows the value of  $M_3(\text{EW})$ , which is the observed gluino pole mass  $M_{\tilde{g}}$  up to loop corrections, for various examples of reduced  $C_3$  studied in Sections 4.1 and 4.2

Case	$A_{\text{UV}}$ (GeV)	$A_{\text{int}}$ (GeV)	$C_3$	$M_3(\text{UV})$ (GeV)	$M_3(\text{EW})$ (GeV)
Case 1.A	$1 \times 10^{16}$	NA	5.9	84	241
Case 1.B	$1 \times 10^{14}$	NA	4.4	98	251
Case 1.C	$1 \times 10^{11}$	NA	2.6	126	274
Case 1.D	$1 \times 10^8$	NA	1.3	181	323
Case 1.E	$1 \times 10^5$	NA	0.4	340	474
Case 2.A	$3 \times 10^{16}$	$5 \times 10^{11}$	0.6	263	106
Case 2.B	$3 \times 10^{16}$	$5 \times 10^{11}$	3.3	113	232
Case 2.C	$4 \times 10^{17}$	$1 \times 10^{14}$	4.3	98	236

#### 4.3. But can the gluino be heavy after all?

One might take the analysis in Sections 4.1 and 4.2, which shows that the relation between SUSY breaking parameters and  $M_Z$  is model dependent, as evidence that the fine-tuning of Eq. (2) can be reduced or removed in some theories. But in the simple examples we have discussed here that is not so—put simply, whatever physics reduces the size of  $C_3$  or  $C_\mu$  through an alteration of the RG evolution of  $M_3$  and  $\mu$  will simultaneously change the predicted superpartner masses such that they remain light. For example, the case described in Fig. 3 shows a possible reduction in  $C_3$  by a factor of almost 10, depending on the scale  $A_I$  at which the intermediate matter is introduced. However, the physical gluino mass in the modified theory is smaller by an amount such that any naturalness bound implies about the same gluino mass in all cases. Explicitly, for the MSSM unified at  $2 \times 10^{16}$  GeV one has  $M_3(\text{EW}) \approx 3M_3(\text{UV})$ , while for the maximum reduction in Fig. 3 one has  $M_3(\text{EW}) \approx 0.43M_3(\text{UV})$ .

We can make this phenomenon even more concrete by revisiting the various examples in Tables 1 and 2 and ask what the implications of varying  $C_3$  are for the low-scale gluino mass. For illustrative purposes let us say that the individual terms in (2) are individually no larger than  $5M_Z^2$ ; for other choices the conclusion is not substantially changed.<sup>8</sup> In particular this choice implies that for each case considered in Tables 1 and 2

we must have

$$M_3(\text{UV}) = \sqrt{5/C_3} M_Z. \quad (29)$$

In Table 3 we have collected the eight cases and computed the implied  $M_3(\text{UV})$  value as well as the resulting electroweak scale value after RG evolution to the scale of  $M_Z$  under the assumptions of each scenario. Even in the most extreme cases considered here a light gluino seems inescapable if we are to avoid large cancellations in (2).

#### 5. Concluding comments

Our goal here has been to take seriously and study phenomenologically the implications of assuming that supersymmetry breaking leads to, and indeed, when the origin of  $\mu$  is included, explains electroweak symmetry breaking. The analysis of the relation between  $\mu$  and  $M_3$  that arises from deriving electroweak symmetry breaking, and thus  $M_Z$ , strongly disfavors the possibility of a robust cancellation between them, at least in the context of the heterotic string-based models we have considered. A similar cancellation among soft parameters is also disfavored. Given the significance of the fine-tuning problem in the electroweak sector of supersymmetric models, additional impetus should be given to the search for models which can provide robust, and not accidental, correlations between soft supersymmetry-breaking terms and the  $\mu$  parameter. In the absence of such a theory it is indeed appropriate to interpret equations such as (3) as implying that the gluino and the parameters which appear in the chargino and neutralino mass matrices should all be truly  $\mathcal{O}(M_Z)$ . Even allowing for a factor of a few to

<sup>8</sup> This choice is roughly equivalent to requiring a “sensitivity” parameter  $\Delta_3 \equiv |(M_3/M_Z^2)\partial M_Z^2/\partial M_3| \leq 10$ , which in earlier days would have been considered an “upper bound” on tolerable amounts of fine-tuning [3,6].

account for possible accidental cancellation it is still the case that in the MSSM gluinos should be at most a few times  $M_Z$  and some charginos and neutralinos lighter than that. With the MSSM values of  $C_3$  and  $C_\mu$  the implied upper limits on gaugino masses are quite small, though not inconsistent with experimental constraints.

The implications of studying how the coefficients can vary in extended theories are also subtle. First, in any reasonable approach within the framework of the MSSM alone, one or more of the coefficients  $C_i$  remain larger than unity, implying some light superpartners exist (always with the caveat that electroweak symmetry breaking is not a coincidence based on accidental cancellations). Perhaps the concerns of the previous paragraph could be partly mitigated because the coefficients, particularly  $C_3$ , were actually reduced by the possible presence of intermediate scale matter below the supersymmetry breaking scale, or simply by beginning the RG evolution of supersymmetry-breaking parameters from a lower scale. But as we saw in the simple approaches considered here, the connection between high and low scale sparticle masses changes in a way correlated with the changes in  $C_3$ , so the actual gluino, chargino, and neutralino masses are not likely to be larger than in the MSSM case.

We can summarize the implications as follows. Several phenomenological clues point to light superpartners, but electroweak symmetry breaking gives our best quantitative constraints. If weak scale supersymmetry is a correct idea, some UV theory generated the soft breaking and  $\mu$  parameters at some higher energy scale. These parameters then resulted in electroweak symmetry breaking, with the measured mass of the  $Z$  as an output. We would prefer to believe that the smallness of  $M_Z$  is not mostly due to large accidental cancellations of parameters which are essentially unrelated at the UV matching scale. As we have shown, this either requires rather light gluinos, charginos, and neutralinos, or it requires a UV theory with special features unlike what we observe in existing frameworks, both stringy and nonstringy. The re-examination of the implications of the supersymmetric radiative electroweak symmetry breaking strengthens the likelihood that superpartners are being produced at the Fermilab Tevatron if supersymmetry indeed provides the explanation of EW symmetry breaking. Similarly, a 500 GeV linear collider should then be above

the threshold to produce lighter charginos and neutralinos.

What would we conclude if superpartners are not detected at the Tevatron? There are four possibilities, the first being that SUSY is simply not the explanation of electroweak symmetry breaking. Only slightly more palatable is to conclude that rather large accidental cancellations actually do occur, or to conclude that the UV theory enforces rather odd relations between  $M_3$  and other parameters. The most acceptable conclusion is likely to be that the Tevatron *did* produce superpartners, but that they were not detected, due to degradation of the discovery signatures as happens in a large number of fairly conventional SUSY scenarios, or to insufficient luminosity to reconstruct a signal. Of course this only re-emphasizes the importance of pursuing aggressive superpartner searches at the Tevatron.

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